Sections 6.4, 6.5, and 6.6: Optimization Problems (Linear Programming)

Steps:

1. Identify what is to be optimized
2. Define Variables
3. Write the system of inequalities
4. Write the objective function
5. Identify the constraints and restrictions
6. Graph the system to find feasible region
7. Identify the vertices of the feasible region
8. Find the optimal solution + verify that it satisfies the constraints.

$-3y + 2x \geq 3$

$-3y \geq 3 + 2x$

$y \leq -1 - \frac{2}{3}x$

INVESTIGATE the Math

p332

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs \$8 to make a racing car and \$12 to make a sport-utility vehicle.

There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.

Steps:

1. Identify what is to be optimized   cost (maximum & minimum)

2. Define Variables
   let r be the # of racing cars
   s be the # of SUVs

3. Write the system of inequalities
   $r \leq 40$ $r + s \geq 70$
   $s \leq 60$

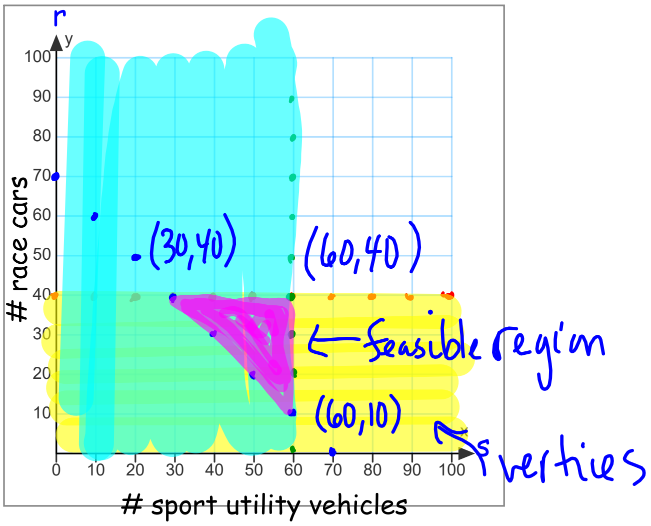
4. Write the objective function (an equation for what is being optimized)
   $C = 8r + 12s$

Objective function: $C = 12s + 8r$

5. Identify the constraints and restrictions

constraints
$r \le 40$
$s \le 60$
$r + s \ge 70$

restrictions
$r \in W$
$s \in W$

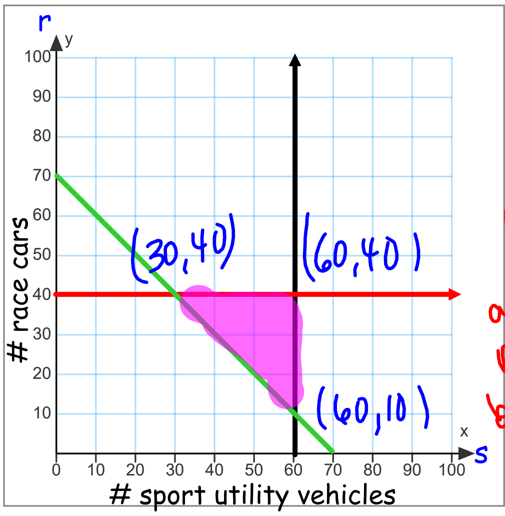


6. Graph the system to find feasible region

Objective function: $C = 12s + 8r$

7. Identify the vertices of the feasible region

8. Find the optimal solution



60 SUVs and 40 race cars will be the max cost of $1040

MAX

(30,40) $C = 12s + 8r$

$C = 12(30) + 8(40)$

$C = 360 + 320$

$C = \$680$

MIN

30 SUVs and 40 race cars will be the min cost of $680

(60,40) $C = 12(60) + 8(40) = 720 + 320 = \$1040$

(60,10) $C = 12(60) + 8(10) = 720 + 80 = \$800$

A refinery produces oil and gas.

- At least 2 L of gasoline is produced for each litre of heating oil.
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.

constraints.

- Gasoline is projected to sell for $1.10 per litre. Heating oil is projected to sell for $1.75 per litre.

← objective function

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

Steps:

1. Identify what is to be optimized revenue (maximum)

2. Define Variables

let h = millions of litres of heating oil

g = millions of litres of gas

3. Write the system of inequalities

$h \leq 9$ $g \geq 2h$

$g \leq 6$

4. Write the objective function

$R = 1.75h + 1.10g$

Objective function: $R = 1.10g+1.75h$

5. Identify the constraints and restrictions

constraints

$g \leq 6$

$h \leq 9$

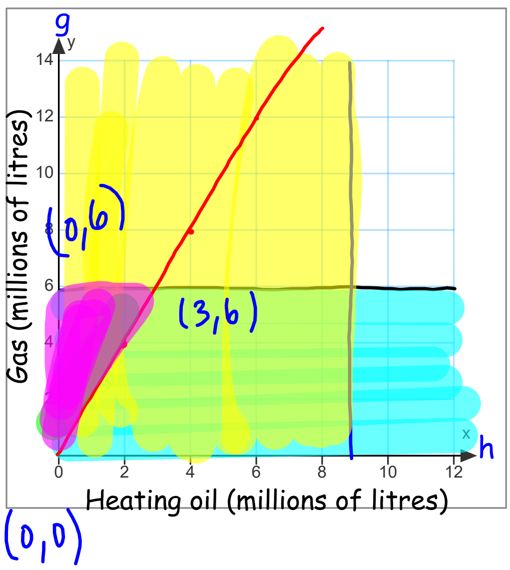
$g \geq 2h$

restrictions

$g \in R, g \geq 0$

$h \in R, h \geq 0$

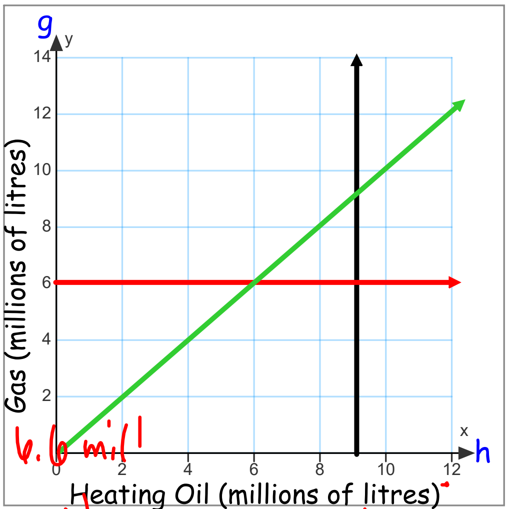
6. Graph the system to find feasible region



Objective function: R = 1.10g+1.75h

7. Identify the vertices of the feasible region

8. Find the optimal solution



(h, g)

(0,0) R = 1.10(0) + 1.75(0) = 0

(0,6) R = 1.10(6 000000) + 1.75(0) = ?? 6.6 mill

(3,6) R = 1.10(6000000) + 1.75(3 000000) = ?? 6.6 + 4.25 mill 10.85 mill

MAX REV => $10.85 mill

TO DO: ① Look over p 329

② CYU p330 (#1 and #2)

A vending machine sells juice and pop. The machine holds at most 240 cans of drink. Sales from the vending machine show that at least 2 cans of juice are sold for each can of pop. Each can of juice sells for $1.00 and each can of pop sells for $1.25. Determine the maximum revenue from the vending machine.

p330 #3

Steps:

1. Identify what is to be optimized Revenue (maximum)

2. Define Variables

   let p be the # of cans of pop sold

   j be the # of cans of juice sold

3. Write the system of inequalities

   $p + j \leq 240$

   $j \geq 2p$

4. Write the objective function

   revenue: $R = 1.25p + 1.00j$

Objective function: R = 1.25p + j

5. Identify the constraints + restrictions

constraints:

$p + j \leq 240$

$j \geq 2p$

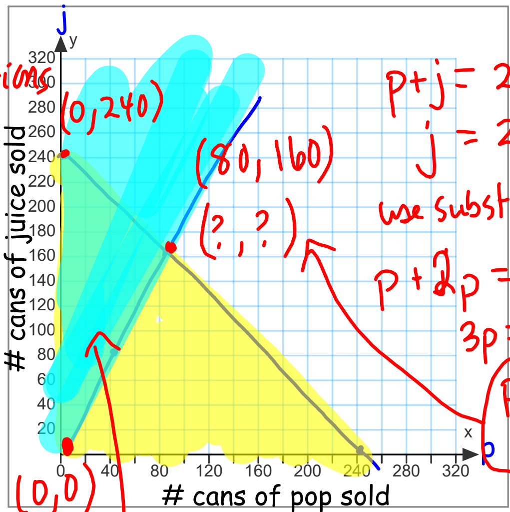
$(y = 2x)$

restrictions

$p \in W$

$j \in W$

6. Graph the system to find feasible region



$p + j = 240$

$j = 2p$

use substitution

$p + 2p = 240$

$3p = 240$

$p = 80$

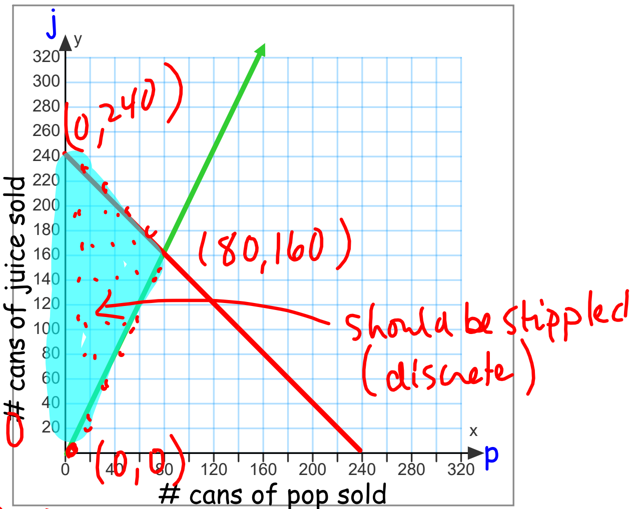
$j = 160$

feasible regions

Objective function: $R = 1.25p + j$

7. Identify the vertices of the feasible region

8. Find the optimal solution



$R = 1.25p + j$

(0,0) $R = 1.25(0) + (0) = 0$

(0,240) $R = 1.25(0) + (240) = 240$

(80,160) $R = 1.25(80) + (160) = 260$

Check that (80,160) satisfies the constraints.

$p + j \leq 240$

$80 + 160 \leq 240$

$240 \leq 240$ ✓

$j \geq 2p$

$160 \geq 2(80)$

$160 \geq 160$ ✓

The max. revenue is \$260 from 80 cans pop 160 can juice.

On a flight between Winnipeg and Vancouver, there are business class and economy seats. At capacity, the airplane can hold no more than 145 passengers. No fewer than 130 economy seats are sold and no more than 8 business class seats are sold. The airline charges $615 for business class seats and $245 for economy seats. What combination of business class and economy seats will result in maximum revenue? What will the maximum revenue be?

p345 #11

Steps:

1. Identify what is to be optimized Revenue (maximum)

2. Define Variables
   let b be the # of business class seats
   e be the # of economy class seats.

3. Write the system of inequalities
   $b + e \leq 145$ $b \leq 8$
   $e \geq 130$

4. Write the objective function
   $$R = 615b + 245e$$

Objective function: R = 615b + 245e

5. Identify the constraints + restrictions

constraints

$b + e \le 145$

$e \ge 130$

$b \le 8$

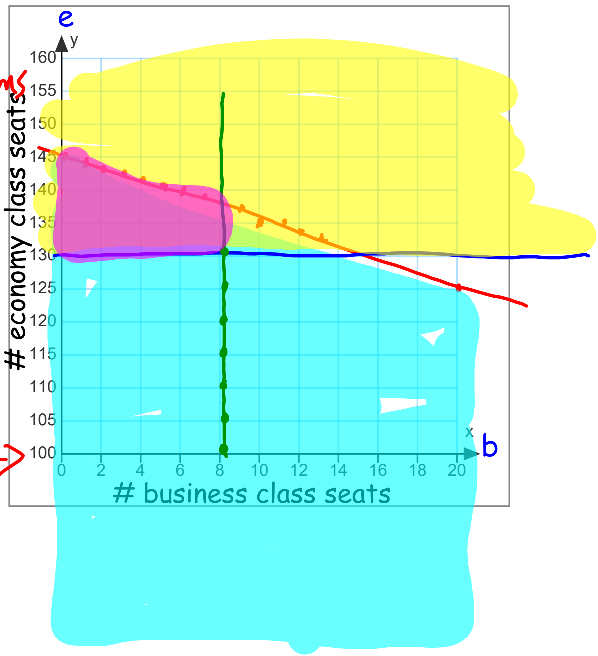
restrictions

$b \in W$

$e \in W$

6. Graph the system to find feasible region

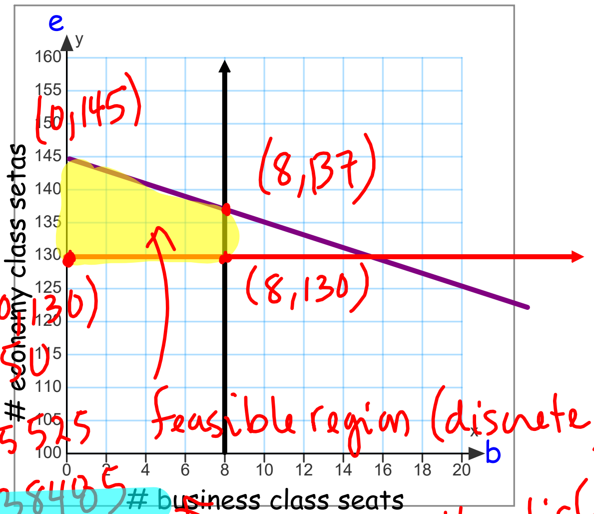
starts at 100



Objective function: $R = 615b + 245e$

7. Identify the vertices of the feasible region

8. Find the optimal solution



(0,145)

(8,137)

(0,130)

(8,130)

feasible region (discrete)

(b,e)	$R = 615b + 245e$
$(0,130)$	$R = 615(0) + 245(13) = 31850$
$(0,145)$	$R = 615(0) + 245(145) = 35525$
$(8,137)$	$R = 615(8) + 245(137) = 38485$
$(8,130)$	$R = 615(8) + 245(130) = 36770$

check that it satisfies inequalities

Write a sentence.

TODO: p331 | 5, 6, 7

p334-335 | 2 + 3

p345 | 12, 13, 14

Practice:

p. 331, Q. 5, 6, 7

p. 334, Q. 2

p. 335, Q. 3

p. 345, Q. 12, 13, 14